Edexcel International AS/A Level

Mathematics

Understanding assessment and improving delivery

First teaching in 2018, first assessment 2019

Pearson Edexcel

Understanding assessment and improving delivery in International A-Level Maths

Wifi Details

Domain:-

Password:-

Please set mobile phones to SILENT

Pearson
Edexcel

Aims and Objectives

- To introduce you to the idea of assessment objectives, what are they and why they are used when writing examination papers

- Analyse recent question papers and learn which types of question match the different assessment objectives

- Investigate different assessment objectives, considering how questions in these areas have been answered by looking at feedback from previous exam series

- Discuss strategies for teaching to try and make sure students can access questions targeting different assessment objectives,

- Review the support Pearson offers for the qualification,

- Network, discuss best practice and share ideas with other teachers.

Pearson Edexcel

Session Agenda

10:00 Introduction and overview

10:10 What are Assessment Objectives ? (AOs)

10:40 Assigning AOs to Edexcel mathematics papers

11:10 BREAK

11:30 Looking at AOs in detail

12:30 Lunch

13:30 Looking at AOs in detail and analysing a recent paper

15:15 Resources and support

15:45 Issues arising

Introduction to Assessment Objectives

Pearson
Edexcel

Problems and examples in this presentation

- Edexcel exam questions undergo a rigorous process before any student sees the examination paper.
  In several slides in this presentation the language and style are not fully that of the exams – indeed there are some problems that would not do at all as exam questions but do have a use as a teaching application.

- The questions themselves are indicative also of the range that students should see in class. They are not intended in any way as a ‘pointer’ to examination questions.
  The Edexcel team have produced material which teachers will be able to use to support their teaching – especially of the new topics.

Pearson Edexcel

Activities in this presentation

There are several activities in this presentation.

Some are as material for delegates to do some mathematics.

In all of the activities delegates are encouraged to consider (with colleagues) such issues as:

Alternative methods of solution
Teaching implications
Demand
How activities/tasks/questions could be adapted.
How well they fit the content
Assigning the Assessment Objective(s)

General Structure of an Assessment

- **Content**
- Facts
- Techniques
- Relationships
- Models

Assessment Objectives

Demonstrate knowledge of facts, techniques and relationships

Demonstrate application of facts, techniques and relationships to solve problems

Demonstrate processes to model real situations and to interpret results of calculations involving models

Pearson Edexcel

General Structure of an Assessment

- Content coverage
  - sufficient for each separate assessment (samples from (nearly) all sections of the content list)
  - complete coverage over a cycle of assessments
- Assessment Objectives
  - **fixed** from assessment to assessment
  - **same weightings** from assessment to assessment (some leeway allowed)

Structure of the Edexcel P1 Assessment

- Content - as given in the specification
- e.g.. Laws of indices for all rational exponents.
- e.g.. Interpret linear and quadratic inequalities graphically.
- e.g.. Solve simultaneous equations; analytical solution by substitution.

Assessment Objectives (AOs)

1. Recall, select and use their knowledge of mathematical facts, concepts and techniques….. **DO**

2. Construct rigorous mathematical arguments and proofs… **PROVE**

3. Recall, select and use their knowledge of standard mathematical models to represent situations in the real world…. **MODEL**

4. Comprehend translations of common realistic contexts into mathematics….. **INTERPRET** results

5. Use contemporary calculator technology and other permitted resources…….. **CALCULATE / FIND**

Pearson Edexcel

Structure of the Edexcel pure units assessment

- All figures in the following table are expressed as marks out of 75.

	AO1	AO2	AO3	AO4	AO5
P1	30–35	25–30	5–15	5–10	1–5
P2	30–35	25–30	5–15	5–10	1–5
P3	30–35	25–30	5–15	5–10	1–5
P4	30–35	25–30	5–15	5–10	1–5

Pearson Edexcel

Structure of the Edexcel applied units assessment

- All figures in the following table are expressed as marks out of 75.

	AO1	AO2	AO3	AO4	AO5
M1	20 – 25	20 – 25	15 – 20	6 – 11	4 – 9
M2	20 – 25	20 – 25	10 – 15	7 – 10	5 –10
S1	20 – 25	20 – 25	15 – 20	5 – 10	5 –10
S2	20 – 25	20 – 25	10 – 15	5 – 10	5 –10
D1	20 – 25	20 – 25	15 – 20	5 – 10	5 –10

Pearson Edexcel

Structure of the Edexcel P1 assessment

- In practice most questions
- on our examination papers have more than one AO assigned to them.

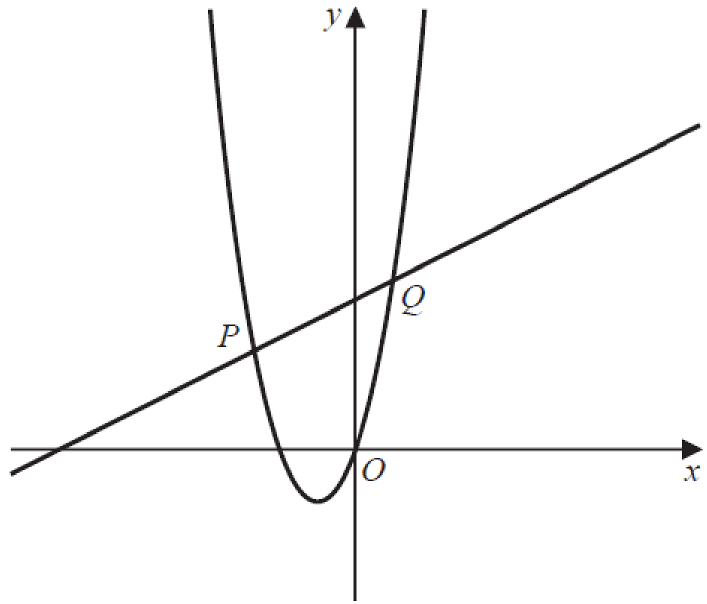


Figure 1

Figure 1 shows a sketch of the curve with equation $y = 2x^2 + 3x$ and the straight line with equation $y = \frac{1}{2}x + 3$

The line meets the curve at the points P and Q, as shown in Figure 1.

AO1 - 3 marks
AO2 - 2 marks
AO4 – 2 marks

(a) Using algebra, find the coordinates of P and the coordinates of Q.

(5)

(b) Hence write down the range of values of x for which $2x^2 + 3x \geqslant \frac{1}{2}x + 3$

(2)

Pearson Edexcel

Activity 1 (Pink)

Initial assignment of AOs to two questions.
Work through each question

The first Q has the associated mark scheme.
Use it to assign the marks to AOs

Then do the second Q

Structure of the Edexcel P1 assessment

- (a)
- $2x^2 + 3x = \frac{1}{2}x + 3$
- $4x^2 + 5x - 6 = 0$
- $(4x - 3)(x + 2) = 0$
- $x = \frac{3}{4}$ or $x = -2$
- $y = \frac{1}{2} \times \frac{3}{4} + 3 = 29/8$ or $y = \frac{1}{2} \times (-2) + 3 = 2$

(b)

$x \leq \frac{3}{4}$ or $x \geq -2$

Pearson Edexcel

Structure of the Edexcel P1 assessment

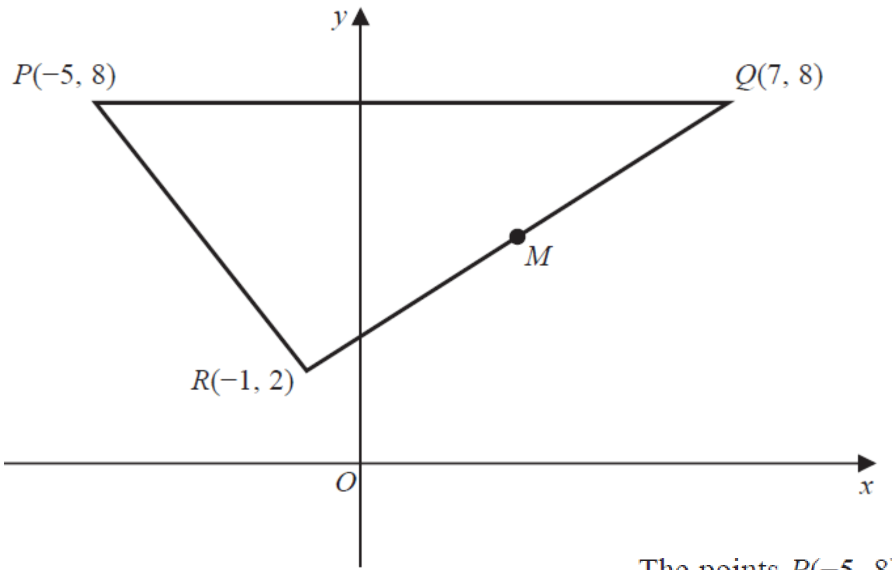


Figure 3

- Decide how to allocate the marks to the AOs
- Which elements of the content does the question require?

The points $P(-5, 8)$, $Q(7, 8)$ and $R(-1, 2)$ form the vertices of a triangle PQR, as shown in Figure 3. The point M is the midpoint of QR.

The line l passes through M and is parallel to PR.

(a) Find an equation for l, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

The line l cuts PQ at the point N.

(b) Find

(i) the coordinates of N,

(ii) the area of triangle MNQ.

(3)

Pearson Edexcel

AO1

Pearson Edexcel

Looking at AO1 on P1, P2, P3 and P4

Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts

Pearson Edexcel

Looking at AO1 on P1, P2, P3 and P4

Knowledge / Recall

Simplify $a \times (\sqrt{a})^{-1}$

Write down an equation of the straight line with gradient 4 passing through P (2, 1).

$y = 3x^4$ Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

Knowledge / Recall

Find the area of a sector radius 3 cm and angle at the centre 0.4 radians

Pearson Edexcel

Looking at AO1 on P1, P2, P3 and P4

- **Concepts**

If $y = x^3$ then $y' = 3x^2$ so $\int x^2\,dx = \frac{x^3}{3} + C$

- **Concepts**

If $ab = 0$ then $a = 0$ or $b = 0$

If $\log_a b = n$ then $a^n = b$

Pearson Edexcel

Looking at AO1 on P1, P2, P3 and P4

- **Techniques**

- Solve quadratic equations using the formula

  Given $y = 48x - 10x^2$ find the maximum value of y

Techniques

Write $\frac{7}{(x-3)(2x+1)}$ as a sum of partial fractions

Activity 2 (Yellow)

Use the sheet for activity 2 to enter some ideas of:

- Knowledge/recall
- Concepts
- Techniques
- from the specification.

Looking at AO1 on P1, P2, P3 and P4

Questions which only assess AO1 are rare:

- e.g.. Specimen Pure 1 Q1 (Differentiation, integration)
- e.g.. Practice Pure 1 Q1 (Transformations)
- e.g.. Specimen Pure 2 Q1a (Binomial expansion)
- e.g.. June 19 Pure 1 Q1 (Integration)
- e.g.. June 19 (C34) Q3 (Parametric to Cartesian)

Pearson
Edexcel

Looking at AO1 on P1, P2, P3 and P4

- e.g.. June 19 (C34) Q3 (Parametric to Cartesian)

3. A curve C has parametric equations

$$x = \sqrt{3}\tan\theta, \qquad y = \sec^2\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{3}$$

The cartesian equation of C is

$$y = \mathrm{f}(x), \qquad 0 \leqslant x \leqslant k, \qquad \text{where } k \text{ is a constant}$$

(a) State the value of k.

(1)

(b) Find $\mathrm{f}(x)$ in its simplest form.

(2)

(c) Hence, or otherwise, find the gradient of the curve at the point where $\theta = \frac{\pi}{6}$

(3)

Pearson Edexcel

Looking at AO1 on P1, P2, P3 and P4

- Activity 3 (Orange)

- Look briefly through Activity 3

- The 3 exam questions were given AO1 only

- Decide on the Knowledge/Concepts/Techniques being assessed
- Do you agree with the assignment of only AO1?

Pearson Edexcel

Looking at AO1 on P1, P2, P3 and P4

- However AO1 appears in most questions:
- -as an explicit part (a)
- - as underlying knowledge/skills/ concepts.

AO1

2. (a) Find $\int \frac{4x+3}{x}\,\mathrm{d}x, \qquad x > 0$

(2)

(b) Given that $y = 25$ at $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(4x+3)y^{\frac{1}{2}}}{x} \qquad x > 0, y > 0$$

giving your answer in the form $y = [\mathrm{g}(x)]^2$.

(5)

AO2

Pearson Edexcel

Looking at AO1 on P1, P2, P3 and P4

As AO1 appears in most questions:

- Students need to be able to have a good understanding of the basics

- as underlying knowledge/concepts/techniques

- in doing so they become more **fluent** in their work

In the next Activity 4 (Yellow) you are asked to think about P3

Activity 4 is a test intended to assess a student's AO1

Are there any significant omissions ?

Are there any questions which do not solely address AO1?

Pearson
Edexcel

AO2

Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.

Pearson
Edexcel

Arguments and Proofs

- ‘Construct rigorous arguments and proofs’

An argument is a series of statements which support a belief/hypothesis.

Arguments can be **deductive** – using the laws of logical reasoning

Or **inductive** – using evidence/observation to support the hypothesis.

(mathematical) proofs are discussed in subsequent slides .

Pearson
Edexcel

A proof must show all **assumptions** you are using, have a clear **sequential list of steps** that logically follow, and must cover **all possible cases**.
You should usually make a **concluding statement**, e.g.. restating the original conjecture that you have proven.

a. Proof by Deduction

This is the simplest type, where you start from known facts and reach the desired conclusion via deductive steps.

An **identity** is an equation that is true for **all values** of the variables. e.g. $x^2 = 4$ is true only for $x = \pm 2$, but $x(x-1) \equiv x^2 - x$ is true for all x.

"Prove that the product of two odd numbers is odd."

?

"Prove that $(3x+2)(x-5)(x+7) \equiv 3x^3 + 8x^2 - 101x - 70$"

?

Proof by Deduction requires you to **start from known facts** and end up at the conclusion. It is **<u>not</u>** acceptable to start with to the conclusion, and verify it works, **because you are assuming the thing you are trying to prove**.

Example: Prove that if three consecutive integers are the sides of a right-angled triangle, they must be 3, 4 and 5.

Incorrect Proof:

?

We are assuming the thing we are trying to prove!

The underlying problem is that this technique doesn't prove there can't be **<u>other</u>** consecutive integers that work – we have only verified 3,4,5 is one such solution.

Correct Proof:

?

We are only assuming things in the 'if' bit. This is fine!

Types of Proof

a. Proof by Deduction

Exam Tip: This is quite a common last part.

Prove that $x^2 + 4x + 5$ is positive for all values of x.

?

Anything squared is at least 0. This is formally known as the '*trivial inequality*'.

Test Your Understanding

Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.

?

Looking at AO2 on P1, P2, P3 and P4

- (a) Show that $(x - 2)$ is a factor of $x^3 - x^2 - x - 2$
- (b) Show that the equation $x^3 = x^2 + x + 2$ has exactly one real root

‘Show’ could be replaced by ‘Prove’.

Looking at AO1 on P1, P2, P3 and P4

(a) Let $P(x) = x^3 - x^2 - x - 2$

$P(2) = 8 - 4 - 2 - 2 = 0$

So, by the <u>factor theorem</u> $(x - 2)$ is a factor of $P(x)$

$$x^3 - x^2 - x - 2 = (x - 2)(x^2 + x + 1)$$

Looking at AO2 on P1, P2, P3 and P4

(b) $x^3 = x^2 + x + 2$

$x^3 - x^2 - x - 2 = 0$

$(x - 2)(x^2 + x + 1) = 0$

So $x = 2$ or $x^2 + x + 1 = 0$

The quadratic has no real roots as the discriminant is – 3

Hence the equation $x^3 = x^2 + x + 2$ has only one real root.

There should be a conclusion to complete.

Pearson
Edexcel

Looking at AO2 on P1, **P2**, P3 and P4

b. Proof by Exhaustion

This means breaking down the statement into **all possible smaller cases**, where we prove each individual case.
(This technique is sometimes known as 'case analysis')

Prove that $n^2 + n$ is even for all integers n.

?

c. Disproof by Counter-Example

While to prove a statement is true, we need to prove every possible case (potentially infinitely many!), **we only need one example to disprove** a statement.
This is known as a **counterexample**.

Disprove the statement:
"$n^2 - n + 41$ is prime for all integers n."

?

Proof By Exhaustion and Counter-example

Not all mathematical statements are amenable to these types of proof

Let $f(n) = n^2 - n + 41$

Then

$f(n)$ is prime for an infinite number of values of n

?

Pearson
Edexcel

Proof (exhaustion) Example

Prove that any square number when divided by 5 leaves a remainder of 0 or 1 or 4

?

?

Proof (exhaustion) Exhaustive sets

- Problems which involve multiplication or division properties by a particular number can often be tackled by using an algebraic exhaustive set.

- For example:

- Show that the square of any whole number is a multiple of 3 or one more than a multiple of 3

An exhaustive set is ?

Squaring gives ?

So any square number will leave a remainder of 0 or 1 when divided by 3

Pearson Edexcel

Proof (exhaustion) Example

SAMs (IAL – P2)
Prove, by exhaustion, that $n^2 + 2$ is not divisible by 4.
(4 marks)

SAMs Solution

n	n^2	n^2+2	
1	**1**	**3**	**Odd**
2	**4**	**6**	**Even**
3	**9**	**11**	**Odd**
4	**16**	**18**	**Even**
5	**25**	**27**	**Odd**
6	**36**	**38**	**Even**

SAMs (IAL – P2)
Prove, by exhaustion, that $n^2 + 2$ is not divisible by 4.

When n is odd, n^2 is odd (odd $\times$ odd $=$ odd) so n^2+2 is also odd	M1
So for all odd numbers n, n^2+2 is also odd and so cannot be divisible by 4 (as all numbers in the 4 times table are even)	A1
When n is even, n^2 is even **and a multiple** of 4, so n^2+2 cannot be a multiple of 4	M1
Fully correct and exhaustive proof. Award for both of the cases above plus a final statement "So for all n, n^2+2 cannot be divisible by 4"	A1*
	(4)

Looking at AO2 on P1, P2, P3 and P4

Disproof by counter-example.

Some further examples

‘Any odd number greater than 1 has an even number of distinct divisors.’

?

?

You can challenge your students to prove that $2^{3n}+1$ is NEVER prime for $n > 0$

Pearson Edexcel

Looking at AO2 on P1, P2, P3 and P4

- A common use of deductive proof is in that of trigonometric identities.

(a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x, \qquad x \neq (2n+1)90°, n \in \mathbb{Z}$$

(3)

?

This solution shows every step so is fully acceptable as a proof

Looking at AO2 on P1, P2, P3 and P4

(a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x, \qquad x \neq (2n+1)90°,\ n \in \mathbb{Z}$$

(3)

$$\frac{1-\cos 2x}{1+\cos 2x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

?

$$\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x \Rightarrow 1-\cos 2x = \tan^2 x\left(1+\cos 2x\right)$$

$$1-\left(1-2\sin^2 x\right) = \tan^2 x\left(1+2\cos^2 x-1\right)$$

$$2\sin^2 x = \frac{\sin^2 x}{\cos^2 x}\left(2\cos^2 x\right)$$

$$2\sin^2 x = 2\sin^2 x$$

?

How many marks should these approaches get?

Looking at AO2 on P1, P2, P3 and P4

Proof (by deduction) Trig identities

- Ideally:
- Start with the more complex side of the identity.
- Simplify the more complex to the less complex side.

?

?

Looking at AO2 on P1, P2, P3 and P4

Proof (by deduction) Trig identities

Proof by deduction requires you to start from **known facts**
What are these?

Knowledge of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$,

and $\sin^2\theta + \cos^2\theta = 1$.

In the Specification for P2

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

As required knowledge for P3

- .....and formulae in the formulae book.

Pearson Edexcel

Looking at AO2 on P1, P2, P3 and P4

Proof (by deduction) Trig identities

Activity 5 (Light Green)
Log and Trig proofs

Activity 5 has examples of attempted proofs of two fairly standard results – one from P2 and one from P3

Look through the proofs and decide whether they are valid or not

Pearson
Edexcel

Proof By Contradiction

✏ To prove a statement is true by contradiction:

- **Assume** that the statement is in fact **false**.
- Prove that this would **lead to a contradiction**.
- Therefore we were wrong in assuming the statement was false, and therefore it must be true.

Prove that there is no greatest odd integer.

? Assumption

? Show contradiction

? Conclusion

How to structure/word proof:

1. "Assume that *[negation of statement]*."
2. [*Reasoning followed by…*] "*This contradicts the assumption that…*" or "*This is a contradiction*".
3. "Therefore [*restate original statement*]."

Pearson
Edexcel

Negating the original statement

The first part of a proof by contradiction requires you to negate the original statement. What is the negation of each of these statements?

"There are infinitely many prime numbers."	"All teachers are clever"	"If it is raining, my garden is wet."
"There are infinitely many non-prime (i.e. composite) numbers."	"There exists a teacher who is not clever."	"If it is not raining, my garden is dry."
"There are finitely many prime numbers."	"No teachers are clever."	"If it is not raining, my garden is wet."
"There are finitely many non-composite numbers."	"The trainer is clever."	"If it is raining, my garden is not wet."

Comments: The negation of "all are" is not "none are". So the negation of "everyone likes green" wouldn't be "no one likes green", but: "not everyone likes green". Do not confuse a 'negation' with the 'opposite'.

Comments: If you have a conditional statement like "*If A then B*", then the negation is "*If A then not B*", i.e. the same condition applies, but the implication is negated.

Pearson Edexcel

Negating the original statement

An important negation which students should be aware of is that of

Original statement	$a < b$
Negation	

Less common is

Original statement	$a = b$
Negation	

More Examples

Prove by contradiction that if n^2 is even, then n must be even.

? Assumption

? Show contradiction

? Conclusion

Remember the negation of "if A then B" is "if A, then not B".

Pearson Edexcel

Looking at AO2 on P1, P2, P3 and P4

Proof on P4

Students should be familiar with the proofs of the infinity of prime numbers and with the irrationality of the square root of 2 In the Spec

It’s then easy to prove, for example that ($\sqrt{2}$ + 1) is irrational – by contradiction

Students don’t find it easy to adapt the proof of the irrationality of $\sqrt{2}$ to, for example, $\sqrt{3}$ – but should be encouraged to do so.

Students should also be encouraged to decide why the ‘proof’ breaks down for $\sqrt{4}$.

Looking at AO2 on P1, P2, P3 and P4

How to structure proof by contradiction (mainly P4)

First stages

Development

Final stages

Proof (by contradiction)

http://www.numberempire.com/numberfactorizer.php

Exploring the proof section

Proof by contradiction –the infinity of primes.

Euclid's illustration of the infinity of primes.

The idea is to consider the sequence which starts

$2 \qquad 2 + 1 \qquad 2 \times 3 + 1 \qquad 2 \times 3 \times 5 + 1 \qquad 2 \times 3 \times 5 \times 7 + 1$

with values

$2 \qquad 3 \qquad 7 \qquad 31 \qquad 211$ which are all prime

so is $2 \times 3 \times 5 \times 7 \times 11 + 1 = 2311$

But thinking of the case $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031$ ($= 59 \times 509$)

So we cannot claim N is always prime.

Pearson Edexcel

More Examples

Prove by contradiction that there are infinitely many prime numbers.

? Assumption

? Show contradiction

? Conclusion

This proof is courtesy of Euclid, and is one of the earliest known proofs.

Looking at AO2 on P1, P2, P3 and P4

As an example here is a slightly different proof to Euclid for the infinity of primes

First stages

Development

Final stages

Looking at AO2 on P1, P2, P3 and P4

‘Proof ‘ in the units – a summary

- P1 – no formal proofs, but ‘show that’s’’ may be set

- P2 – 1.1 (deductive proof),1.2 (proof by exhaustion) and 1.3 (Disproof by counter example)

- P3 – 2.2 and 2.3 (proofs of trigonometric identities)

- P4 – 1.1 (proof by contradiction)

Pearson Edexcel

Looking at AO2 on P1, P2, P3 and P4

‘Activity 6 (Light Blue) – Proof

Activity 6 contains 10 proofs

Select any that interest you and attempt a proof.

What knowledge/ concepts and techniques are required?

Pearson
Edexcel

Looking at AO2 on P1, P2, P3 and P4

‘Extended arguments involving the manipulation of mathematical expressions.’

Stage 1

↓

Stage 2

↓

Stage 3

↓

Stage 4

Translation and understanding

Calling upon relevant knowledge

↓ ↑

Planning a strategy

Execution and interpretation

Looking at AO2 on P1, P2, P3 and P4

‘Extended arguments involving the manipulation of mathematical expressions.’

Solving extended problems is hard – especially in a limited time!

The difficulty of a problem depends upon several factors!

Pearson
Edexcel

Looking at AO2 on P1, P2, P3 and P4

‘Extended arguments involving the manipulation of mathematical expressions.’

…..whether the answer is given or not:

‘Show that…’ instead of

‘Find’

For an argument supporting reasons do not usually have to be given

We could claim that ‘Show that’ is easier than ‘Find’ because it gives the student a definite end point

We could claim that ‘Show that’ is harder than ‘Find’ because it could force the student to use a specific method.

Scaffolding

Scaffolding is the term used to add structure to a question which will usually require extended mathematics .

$$f(x) = \frac{6 - 5x - 4x^2}{(2 - x)(1 + 2x)} \qquad x > 2$$

Prove that f(x) is a decreasing function.

Just think for a moment about what strategies would students plan to use to answer this question……

The question was eventually scaffolded as………

Pearson
Edexcel

Scaffolding

Scaffolding is the term used to add structure to a question which will usually require extended mathematics to answer it.

Gives a start. Makes it clear what is required

$$\frac{6 - 5x - 4x^2}{(2 - x)(1 + 2x)} \equiv A + \frac{B}{(2 - x)} + \frac{C}{(1 + 2x)}$$

(a) Find the values of the constants A, B and C.

$$f(x) = \frac{6 - 5x - 4x^2}{(2 - x)(1 + 2x)} \qquad x > 2$$

(b)

(c)

Pearson Edexcel

Scaffolding

As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

A curve has equation

$$y = \ln(1 - \cos 2x), \qquad x \in \mathbb{R},\ 0 < x < \pi$$

find the exact coordinates of the point on the curve where $\dfrac{dy}{dx} = 2\sqrt{3}$

No scaffolding.
This Q could be classed as basically all AO2

Pearson
Edexcel

Looking at AO2 on P1, P2, P3 and P4

:As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

A curve has equation

$$y = \ln(1 - \cos 2x), \qquad x \in \mathbb{R},\ 0 < x < \pi$$

Show that

(*a*) $\dfrac{dy}{dx} = k \cot x$, where k is a constant to be found.

Hence find the exact coordinates of the point on the curve where

(*b*) $\dfrac{dy}{dx} = 2\sqrt{3}$

Looking at AO2 on P1, P2, P3 and P4

As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

(ii) (*a*) Use the substitution $x = \sec\theta$ to show that

$$\int_1^2 \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x = \int_0^{\frac{\pi}{3}} \tan^2\theta\,\mathrm{d}\theta$$

(*b*) Hence find the exact value of

$$\int_1^2 \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x$$

Pearson Edexcel

Looking at AO2 on P1, P2, P3 and P4

As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

find the exact value of

Looking at AO2 on P1, P2, P3 and P4

Questions assessing AO2 are almost always assigned AO1 marks also. This is because the processes and knowledge required to solve a complex problem are based on AO1

We can see this by looking at a particular topic from the Specification.

To see how this works we'll look at **parameters**.

Looking at AO2 on P1, P2, P3 and P4

The scheme of work – on the website – gives some further details

3.1	Parametric equations of curves and conversion between cartesian and parametric forms.	
5.1	Differentiation of simple functions defined implicitly or parametrically.	The finding of equations of tangents and normals to curves given parametrically or implicitly is required.
6.1	Evaluation of volume of revolution.	$\pi \int y^2 \, \mathrm{d}x$ is required, but *not* $\pi \int x^2 \, \mathrm{d}y$. Students should be able to find a volume of revolution, given parametric equations.

P4 Parameters – how AO1 and AO2 work together (also with AO4)

3.1	Parametric equations of curves and conversion between cartesian and parametric forms.

From a Principal Examiner Report

………only a minority of students were able to use one of the trigonometric forms of Pythagoras to eliminate *t* and manipulate the resulting equation to obtain an answer in the required form.

$x = 6\cos 2t$, $y = 2\sin t$

$x = \tan t$, $y = 2\sin^2 t$

$x = 1 + \sqrt{3}\tan t$, $y = 5\sec t$

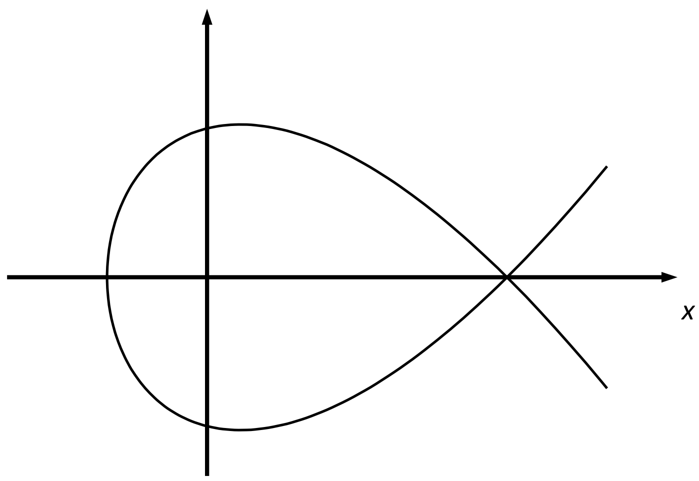
$x = 3\cos t$, $y = 9\sin 2t$

$x = \tan t$, $y = 2\sin^2 t$

$x = 8\cos^3 t$, $y = 6\sin^2 t$

These were taken from recent C34 papers. Most involve use of $\sin^2 + \cos^2 = 1$ or its equivalent.

P4 Parameters – how AO1 and AO2 work together (also with AO4)



$x = 2t^2 - 2, \;\; y = t(t^2 - 4)$

Find the coordinates of points where
- the curve crosses the axes
- crosses the line $x = 2$

P4 Parameters – how AO1 and AO2 work together (also with AO4)

e.g

5.1	Differentiation of simple functions defined implicitly or parametrically.	The finding of equations of tangents and normals to curves given parametrically or implicitly is required.

x = 6 cos2*t*, y = 2sin*t*

Show that y' = λ cosec t, giving the exact value of the constant λ.

x =1 + √3 tan *t*, y = 5sec *t*

Show that y'= λ sin t, giving the exact value of the constant λ.

x = 8cos^3 *t*, y = 6sin^2 *t*

It's almost always easier to use $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$

This also can involve use of trig identities to achieve a given answer

Pearson Edexcel

P4 Parameters – how AO1 and AO2 work together (also with AO4)

e.g.. Parameters

$x = 8\cos^3 t$, $y = 6\sin^2 t$

Find the equation of the normal to the curve at $t = \pi/3$
Give the equation in the form $ax + by = c$

Pearson
Edexcel

P4 Parameters – how AO1 and AO2 work together (also with AO4)

6.1	Evaluation of volume of revolution.	$\pi \int y^2 \;\mathrm{d}x$ is required, but *not* $\pi \int x^2 \;\mathrm{d}y$. Students should be able to find a volume of revolution, given parametric equations.

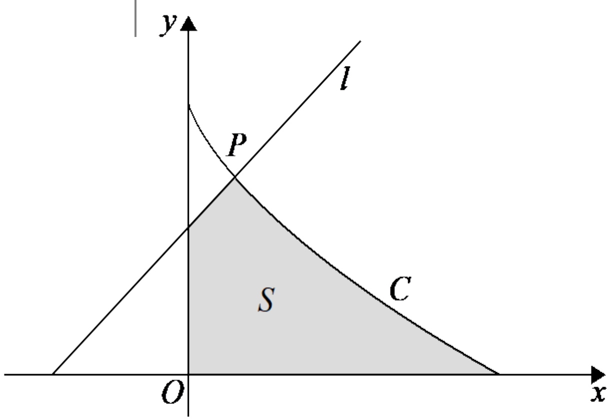


Figure 6

$x = 8\cos^3\theta$, $y = 6\sin^2\theta$,

The line l is the normal to C at P with $t = \pi/3$

(*c*) Show that the area of *S* is given by

$$4 + 144\int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta - \sin\theta\cos^4\theta)\,\mathrm{d}\theta$$

(*d*) Hence, by integration, find the exact area of *S*.

So students have to be able to transform an integral in x to one in θ

Pearson Edexcel

AO3

Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.

Looking at AO3 on P1, P2, P3 and P4

AO3 refers to the use of standard models – this means models that are commonly known or models that are straightforward to understand.

Here is an example from the material in P1

At 12:00 a ship is at a point A 48 km West of a port P.
The ship sails on a bearing of 060^{o} to a point B.
PB = 36 km

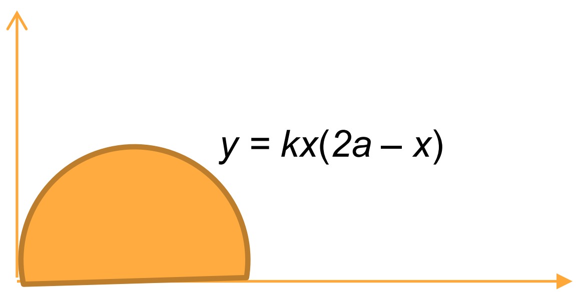
Find by calculation, the two possible bearings of B from P.
Give your answers correct to the nearest degree.

Pearson Edexcel

Looking at AO3 on P1, P2, P3 and P4

AO3 refers to the use of standard models – this means models that are commonly known or models that are straightforward to comprehend.

Here is an example from the material in P1



The diagram represents the cross-section of a tunnel.
The width of the cross section is 10 m and the height is 6 m

(a) Find the value of k and the value of a.
(b) Use integration to find the area of the cross-section of the tunnel.

Pearson Edexcel

Looking at AO3 on P1, P2, P3 and P4

AO3 refers to the use of standard models – this means models that are commonly known or models that are straightforward to comprehend.

Common models on P2 require the use of Arithmetic or Geometric series for describing growth and decay.

Key techniques involve:

Pearson
Edexcel

Looking at AO3 on P1, P2, P3 and P4

Here is an typical example from the material in P3

A rare species of mammal is being studied. The population P, t years after the study started, is modelled by the formula

$$P = \frac{900e^{\frac{1}{4}t}}{3e^{\frac{1}{4}t} - 1}, \qquad t \in \mathbb{R}, \qquad t \geq 0$$

Students should be able to:

Looking at AO3 on P1, P2, P3 and P4

Population mathematics in P3

Students need to be able to differentiate expressions of the form $P = \frac{K}{1+Ae^{-\gamma t}}$

Or equivalently $P = \frac{Ke^{\gamma t}}{Ce^{\gamma t}+D}$

A more complex model is $N = \frac{Re^{yt}}{S + Te^{\sigma t}}$

Not a case where the product rule for differentiation should be used.

Looking at AO3 on P1, P2, P3 and P4

Here is another example from the material in P3

Students need to be able to use the formula
$y = a \cos \acute{\omega} t + b \sin \acute{\omega} t + c$
to describe oscillatory motion.

On P2 this could be set as a single sine or cosine function

This usually requires students set up and to solve equations such as $5 = 6 \cos \acute{\omega} t + 3 \sin \acute{\omega} t + 2$

Looking at AO3 on P1, P2, P3 and P4

Here is an example from the material in P4

Students need to understand modelling using differential equations.
The key idea is that $\frac{\mathrm{d}A}{\mathrm{d}t}$ represents the rate of change of the quantity *A*

Overlaps with AO4

Often the form of the rate of change is given

Students do need to be able to write down/derive or interpret a D.E. which shows the rate to change of *A* to be:

- constant (with interpretation of a negative sign)
- to be proportional to *A*.

Pearson Edexcel

AO4

Pearson
Edexcel

Looking at AO4 on P1, P2, P3 and P4

Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.

Overlaps with AO3

.

'Hence' is often a key word in AO4

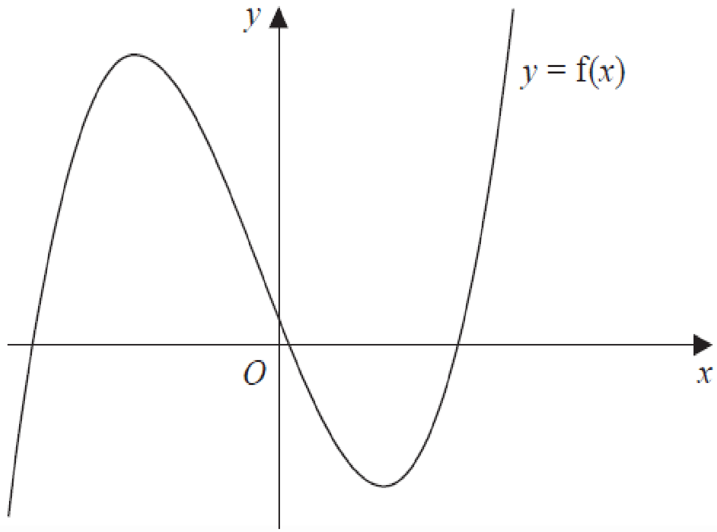
'Deduce' is often a key word in AO4

Pearson
Edexcel

Looking at AO4 on P1, P2, P3 and P4

Here is an example from the material in P1

4.



....use the results of calculations to make predictions......

.

$$f(x) = 2x^3 + \frac{3}{2}x^2 - 18x + 3$$

(a) Find the set of values of x for which f(x) is decreasing

(b) Hence find the number of roots of the equation f(x) = k (k constant) according to the values of k

Pearson
Edexcel

Looking at AO4 on P1, P2, P3 and P4

Here is an example from the material in P3

….use the results of calculations to make predictions……

(a) Prove that $\tan x + \cot x \equiv 2\operatorname{cosec} 2x$ for $x \neq n\pi/2$

(b) Deduce that the equation $\tan x + \cot x = 1$ has no real solutions.

Looking at AO4 on P1, P2, P3 and P4

Here is an example from the material in P4

'..use the results of calculations to make predictions.'

A curve has equation

$$y = \ln(1 - \cos 2x), \qquad x \in \mathbb{R},\ 0 < x < \pi$$

Show that

(*a*) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = k \cot x$, where k is a constant to be found.

(4)

Hence find the exact coordinates of the point on the curve where

(*b*) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\sqrt{3}$

(4)

Looking at AO4 on P1, P2, P3 and P4

Here is an example from the material in P2

'….read critically'

Solve the equation $2\log_5(2y + 1) - \log_5(2 - y) = 1$

explaining clearly why there is only one real solution

Solve the equation

$$\log_4\left(5x^2 - 11\right) = \log_2\left(3x - 5\right).$$

x real

Looking at AO4 on P1, P2, P3 and P4

Here is an example from the material in P3

'…read critically and comprehend longer mathematical arguments…

Solve $\cos 2x + \sin 2x = \frac{1}{\sqrt{2}}$ for values of x in the interval $0 \le x \le 2\pi$

(1) $\cos 2x + \sin 2x = \sqrt{2}\cos(2x + \frac{\pi}{4})$

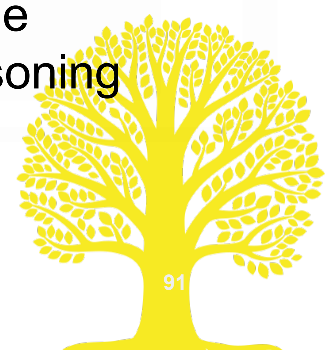
(2) So the equation becomes $\cos(2x + \frac{\pi}{4}) = \frac{1}{2}$

(3) A solution of $\cos\theta = \frac{1}{2}$ is $\theta = \frac{\pi}{3}$

(4) So $2x + \frac{\pi}{4} = \frac{\pi}{3}$ and $x = \frac{\pi}{24}$

(5) The second solution is $2\pi - \frac{\pi}{24} = \frac{23\pi}{24}$

In which lines is the mathematical reasoning <u>incorrect</u>?



Activity 7 (Red)

Activity 7 asks you to work through the 3 previous examples.

What other questions could be set which have the same issues as the log questions?

There are various issues with setting a question like the previous slide.
Please identify at least one.

AO5

Pearson
Edexcel

Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.

Pearson
Edexcel

Use contemporary calculator technology…… accurately and efficiently

Which calculators are allowed in Edexcel examinations.?

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

So, some models of graphical calculators are permitted.

Pearson Edexcel

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

From the P1 specification

Solution of quadratic equations using the formula
Mensuration and radian measure

Graphs:
Quadratic
k/x and k/x^2
sin, cos and tan

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

From the P2 specification

solution of quadratic inequalities (sum of an Arithmetic Series)
solution of inequalities requiring taking logs (sum of a Geometric Series)

values of binomial coefficients

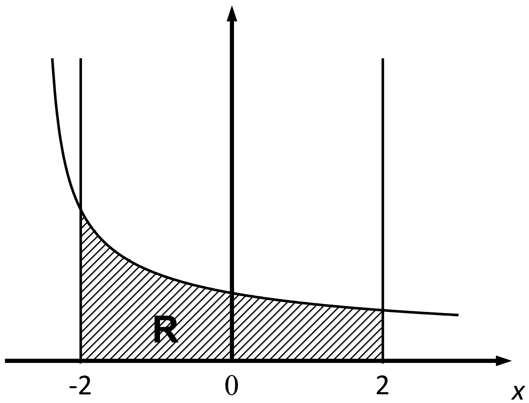
solution of trig equations – degrees and radians

evaluation of expressions after integration
trapezium rule*

*The derivation of the formula is not required knowledge – but should – at least informally – be shown

Trapezium rule

8.3 Use of increasing number of trapezia to improve accuracy and an estimate of the error may be required.



The figure shows a sketch of part of the curve C, with equation

$$y = \frac{1}{\sqrt{2x+5}}$$

The finite region **R** shown shaded is bounded by C, the *x*-axis and the lines *x* = ±2

x	-2	-1	0	1	2
$y = \frac{1}{\sqrt{2x+5}}$	1		0.4472		0.3333

(a) Complete the table.

(b) Use the trapezium rule to find an estimate of the area of **R**.

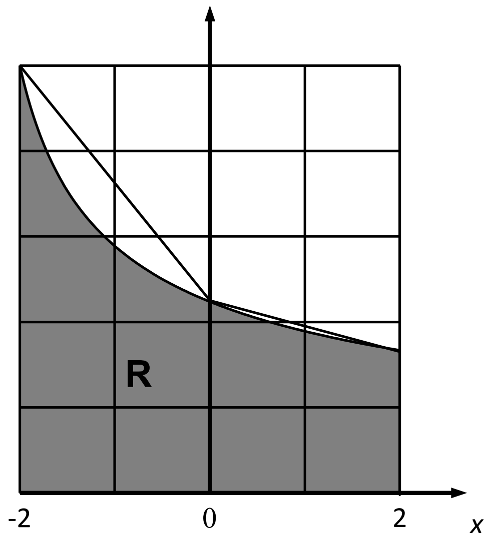
(c) Given that the exact area of **R** is 2, work out an estimate of the error.

In the exam the language is much more precise!

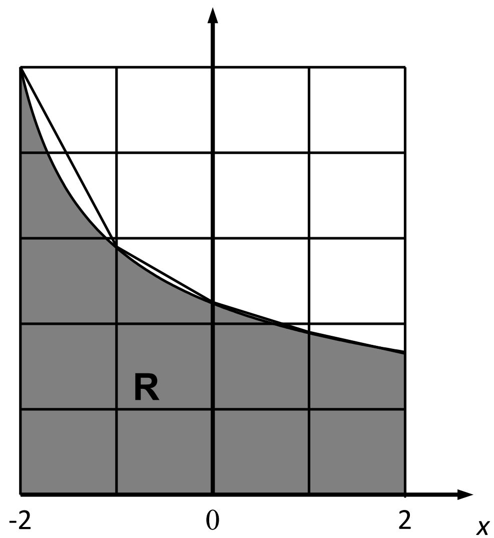
Pearson Edexcel

Trapezium rule

8.3 Use of increasing number of trapezia to improve accuracy and an estimate of the error may be required.



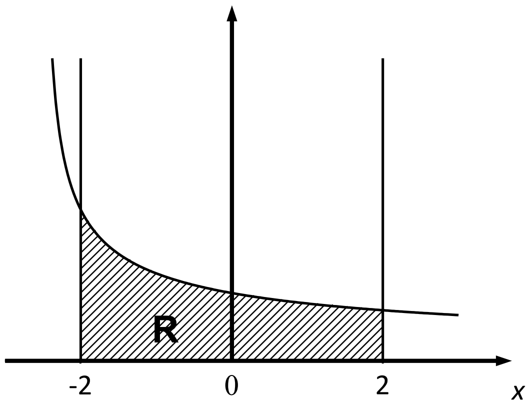
2 trapeziums



4 trapeziums

Trapezium rule

8.3 Use of increasing number of trapezia to improve accuracy and an estimate of the error may be required.



x	y	x	y	x	y
-2	1	-2	1	-2	1
0	0.447214	-1	0.57735	-1.5	0.707107
2	0.333333	0	0.447214	-1	0.57735
	2.227761	1	0.377964	-0.5	0.5
		2	0.333333	0	0.447214
			2.069195	0.5	0.408248
				1	0.377964
				1.5	0.353553
				2	0.333333
					2.019052

In this case doubling the number of strips from 4 to 8 reduces the error from about 3½% to about 2%

If you are faced with the question does doubling the number of strips half the error, how would you answer it?

Geometrically more strips leads to a lower error because ………

However, we must be careful when increasing the number of strips because…..

Pearson Edexcel

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

From the P3 specification

Solution of trig equations

Straight line graphs derived from data of the form $y = ax^n$ or $y = kb^x$

Behaviour of f(t) when t gets large. (exponential type models)

*Location of roots of $f(x) = 0$ by looking for sign changes

*Iteration

Graphs $y = |ax + b|$

Pearson Edexcel

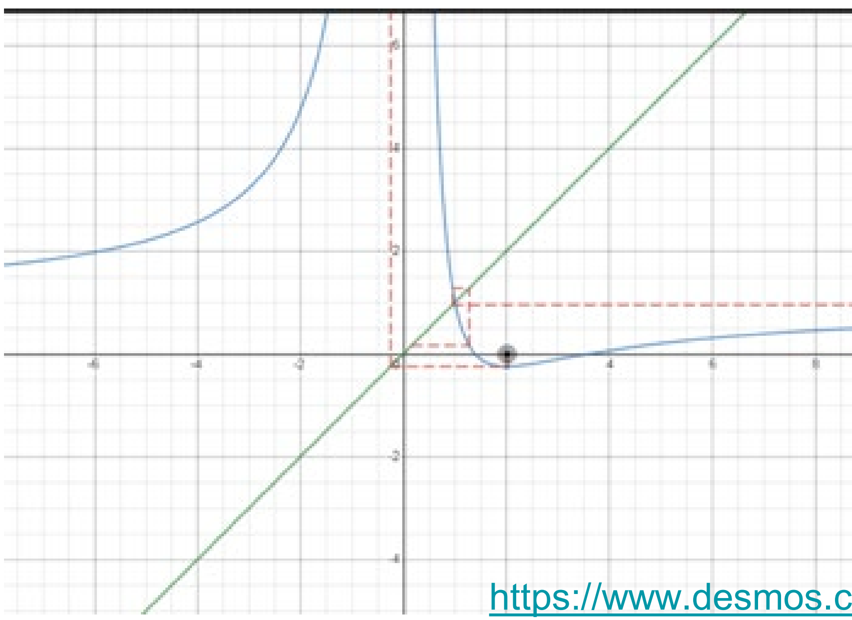
Iteration

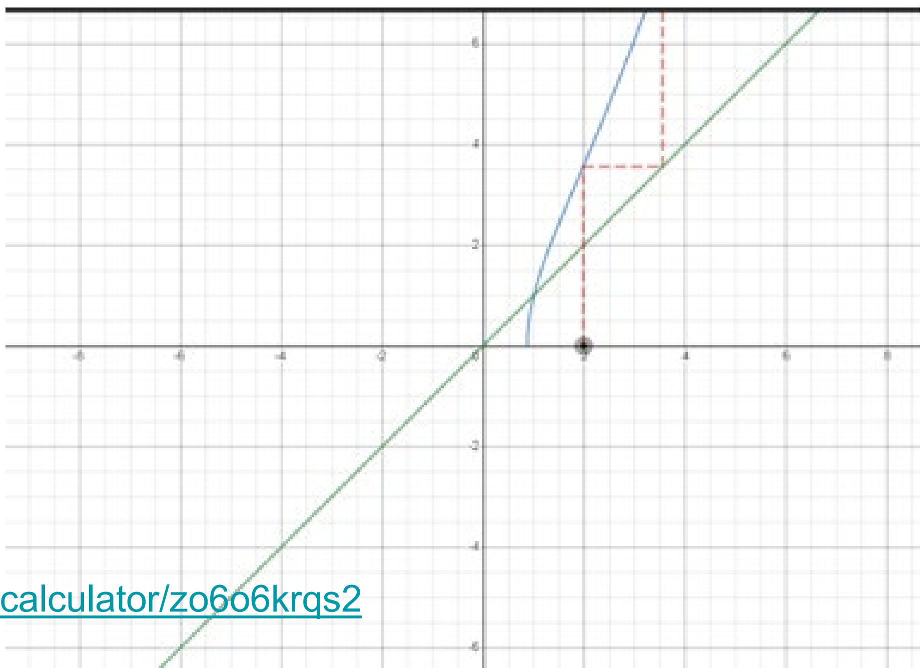
Use contemporary calculator technology…… accurately and efficiently

Students will be given the formula to use
Students should be aware that not all rearrangements of an equation lead to a convergent sequence.

But not be assessed on this

Two different rearrangements of the equation $x^3 - x^2 + 5x = 5$





https://www.desmos.com/calculator/zo6o6krqs2

$$x = \frac{x^2 - 5x + 5}{x^2}$$

$$x = \sqrt{x^3 + 5x - 5}$$

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

From the P4 specification

Nothing explicit - finding angles between vectors

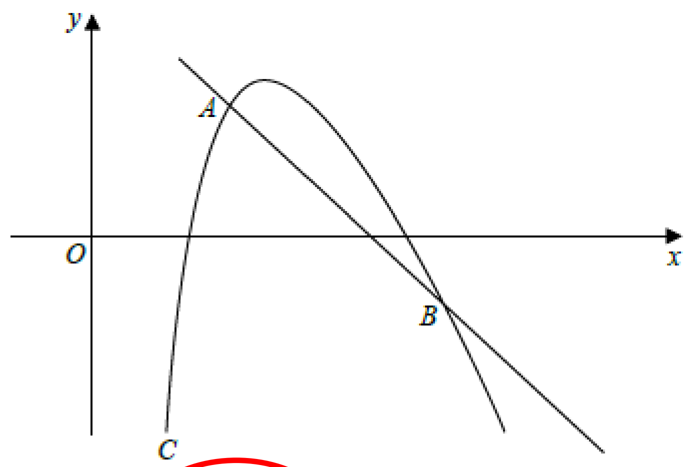
Pearson
Edexcel

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

Consequences:

Specific wording in questions **to prevent** the use of calculators



(b) Use algebra to find the coordinates of B.

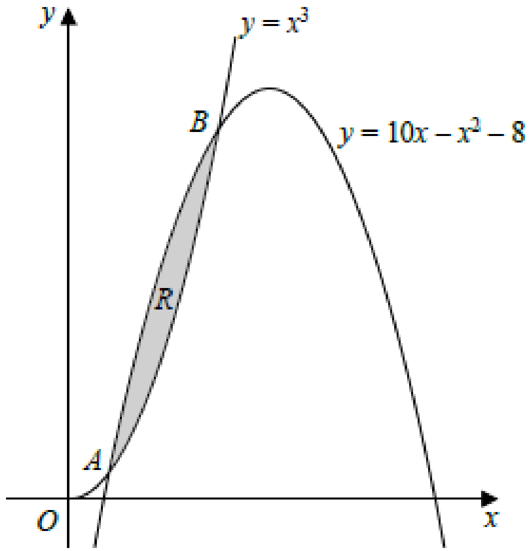
Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

Consequences:

Specific wording in questions to prevent the use of calculators

8.



(b) Use algebra to find the coordinates of the point B

(c) Use calculus to find the exact area of R

Pearson
Edexcel

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

Consequences:

Specific wording in questions to **prevent** the use of calculators

(ii) Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} \sin 2x + \sec\frac{1}{3}x\tan\frac{1}{3}x \, dx$$

(b) Use calculus to find the coordinates of A.

(ii) Find $\int_1^2 f(x)\,dx$, giving your answer in the form $a + \ln b$, where a and b are constants.

Find, by integration, the exact value for the area of R.

Give your answer in terms of $\ln 2$

Pearson
Edexcel

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

Consequences:

Specific wording in questions to **prevent** the use of calculators

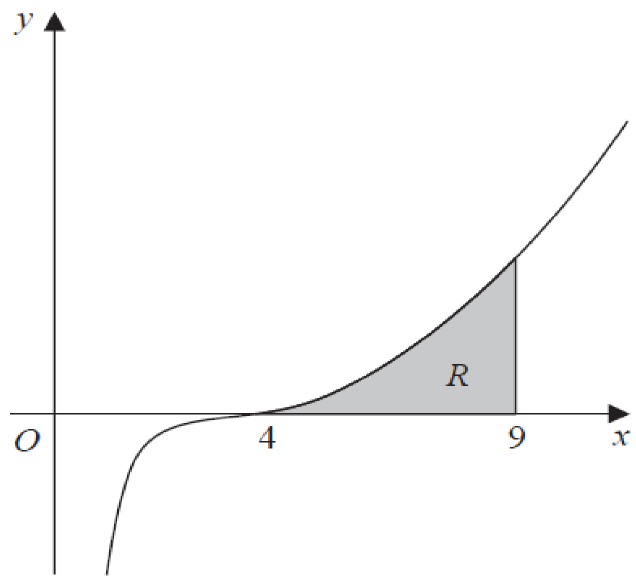


Figure 1

Find the exact area of R.

In this question you must show all steps of your working.

Solutions relying on calculator technology are not acceptable.

Pearson Edexcel

Looking at AO5 on P1, P2, P3 and P4

…and other permitted resources (such as formulae booklets…)

From the P1 specification

One form of the Cosine Rule

Pearson
Edexcel

Looking at AO5 on P1, P2, P3 and P4

…and other permitted resources (such as formulae booklets…)

From the P2 specification

*n*th terms and sums of arithmetic and geometric series

Change of base rule for logs

Binomial series (both forms)

Trapezium rule.

Students have to learn

Looking at AO5 on P1, P2, P3 and P4

…and other permitted resources (such as formulae booklets…)

From the P3 specification

Trig identities
e.g.. sin (A + B) = sin A cos B + sin B cos A

Students have to learn

Looking at AO5 on P1, P2, P3 and P4

…and other permitted resources (such as formulae booklets…)

From the P4 specification

Students have to learn

Scalar product of 2 column vectors

Pearson
Edexcel

Looking at AO5 on P1, P2, P3 and P4

……give answers to appropriate accuracy.

From the P3 specification

By looking at the signs of $f(a + \varepsilon)$ and $f(a - \varepsilon)$

$f(3.18) = 0.01$ and $f(3.19) = -0.7$

There is a root in the interval $(3.18, 3.19)$

Pearson Edexcel

Looking at AOs on P1, P2, P3 and P4

Activity 8 (Pink)

Use the edited copy of June 2019 paper 2 together with the Mark scheme..

Assign AOs (1 to 5) to the questions where appropriate.

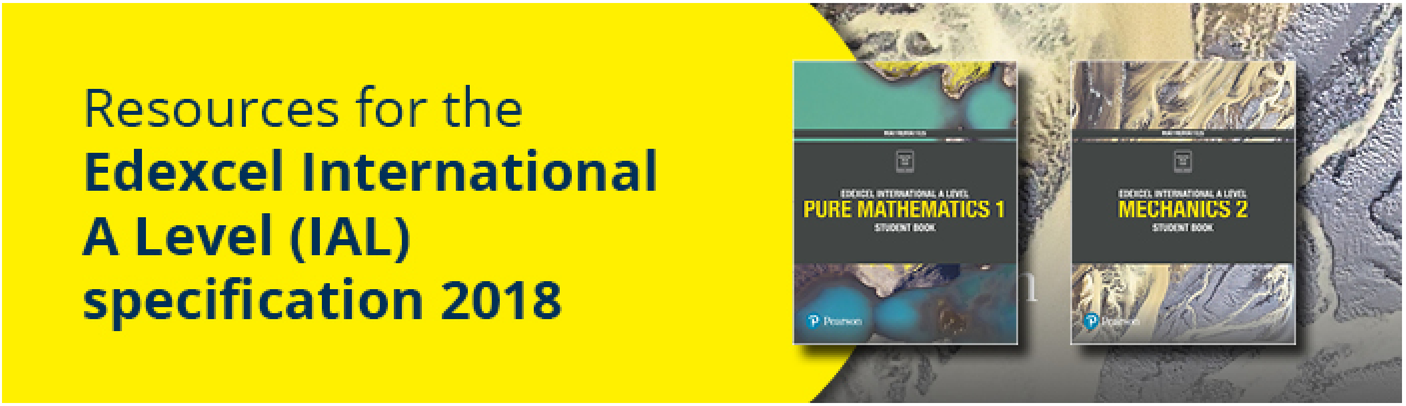
Remember your targets are

	AO1	AO2	AO3	AO4	AO5
P1	30–35	25–30	5–15	5–10	1–5
P2	30–35	25–30	5–15	5–10	1–5
P3	30–35	25–30	5–15	5–10	1–5
P4	30–35	25–30	5–15	5–10	1–5

Resources and Support

Pearson Edexcel

Pearson Publishing



Published resources for each of the 14 Units for International Mathematics

ResultsPlus

- Free online results analysis tool for teachers.
- Provides a detailed breakdown of student performance in Pearson Edexcel exams.
- Identify topics and questions where the student could benefit from further learning and inform teaching strategies and approaches.
- Benchmark your school's performance against other Pearson Edexcel schools in your country.
- Not just a post-results tool: Mock exam results can also be fed into the system to produce analysis.
- Find student results analysis from their previous Pearson Edexcel school.
- ResultsPlus Direct gives your students access to their final grades and performance breakdown, wherever they are.
- Schools can sign up for free ResultsPlus account in just a few quick and easy steps: https://qualifications.pearson.com/en/support/Services/ResultsPlus.html

Pearson Edexcel

examWizard

- A free tool for teachers which helps you make quick homework assignments, topic tests and mock exams.
- Questions tagged against unit, topic and assessment objective or simply choose a whole past paper.
- Use existing mark schemes for accurate marking.
- Use examiner report for insight.
- Most recent exam content available sooner.
- Use the results to understand where students need more support, informing teaching strategies.

Pearson
Edexcel

New Access to Script (ATS) Online Portal

Access to Scripts (ATS) is a free online portal which allows teachers to immediately access electronically marked exam papers

Provides enhanced transparency and

- Offers transparent approach to marking process
- Provides better understanding of marking before requests for enquiries about results are made
- Provides excellent aid for teaching and preparing other cohorts for examinations by helping you to evaluate a student’s performance on particular questions in relation to what they have been taught.

Available instantly from results day for all our examination series, for a defined window, you can view and download scripts which have been marked online free of charge from our Self-Service Portal.

For more information on ATS, and the post results windows, visit our post-results pages.

Pearson
Edexcel

Contact your dedicated Subject Advisor

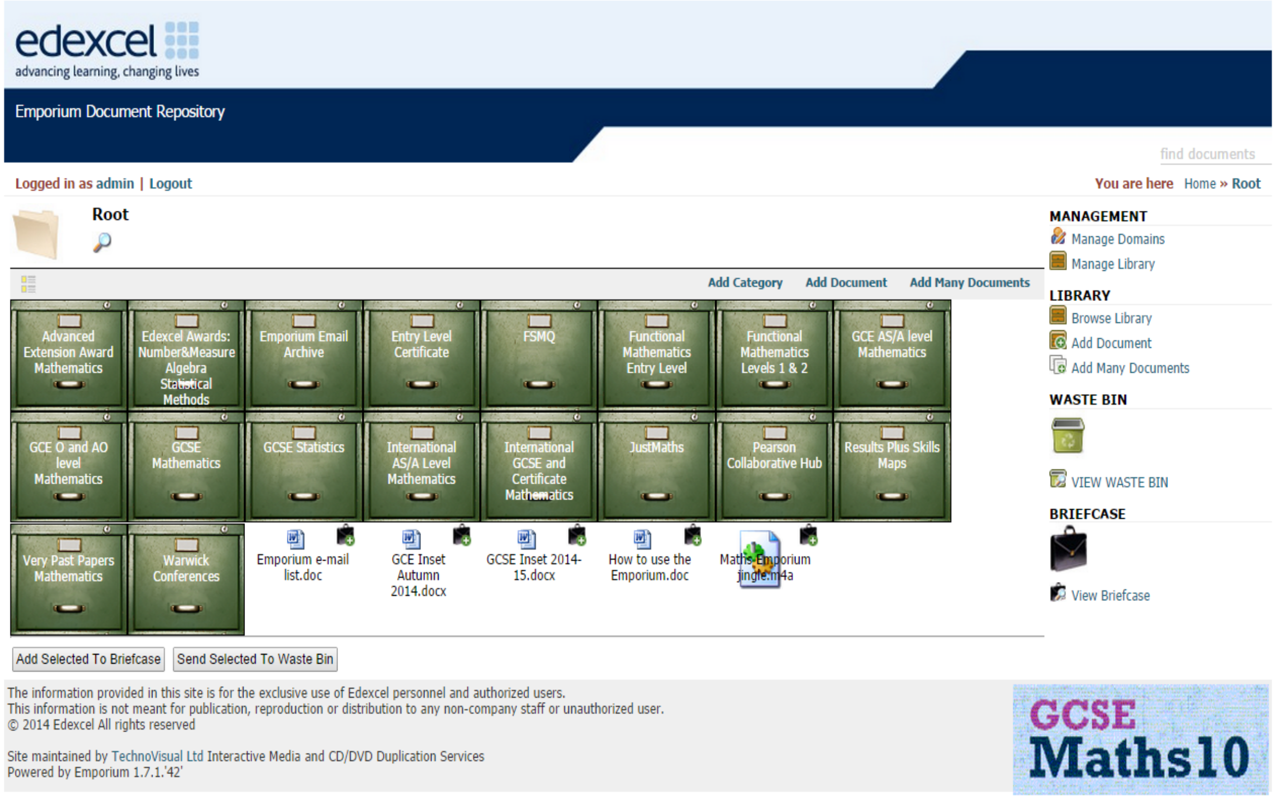
Subject Advisor details

Your subject advisor is **Graham Cumming**
Phone: **+44 (0)20 70102174**
Twitter: **@EmporiumMaths**
Email: TeachingMaths@pearson.com

Sign up for monthly newsletters from Graham to stay on top of qualification updates, training, course materials and industry news.

Mathematics Emporium

- Website at www.edexcelmaths.com



- Email updates from mathsemporium@pearson.com

Other useful links

1. **Grade Boundaries**

This page shows the minimum marks needed to achieve a certain grade for all UK and international examinations. Also refer to the examiners report which is available for download with other documents.

2. **Examination Results Statistics**

Results statistics summarise the overall grade outcomes of candidates sitting Pearson Edexcel examinations.

3. **Progress to University**

Here you can find information and guidance about how to progress to universities worldwide with Pearson Edexcel qualifications.

4. **Access to scripts**

Make an informed enquiry about results (EARs) using our free access to scripts portal.

Pearson International Schools Community

Connect with international teachers around the world

- Connect with other teachers working in international schools and join groups who have shared interests, subjects or location
- Read topical news and articles and share yours
- Advertise jobs at your school or find job opportunities
- Download free resources
- Sign up for events.

Sign up today at:

- pearson.com/internationalschools/blog.

Pearson
Edexcel

Any questions?

Thank you for attending this event.

How did we do?
Please fill in the evaluation form that you'll receive via e-mail in a few minutes.

ALWAYS LEARNING